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MAINTENANCE OF THE NEAR-WALL CYCLE OF TURBULENCE FOR HYBRID RANS/LES OF FULLY-DEVELOPED CHANNEL FLOW

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Abstract
Hybrid RANS/LES modeling of near-wall turbulence is investigated for fully-developed turbulent channel flow under very coarse resolution, a resolution not resolving the longitudinal eddies of the buffer layer in near-wall flow but not coarse enough to encompass an "effective" ensemble of eddies to give RANS. Without bursting from the buffer layer, the partially-resolved turbulence is suppressed. We model the effects of the buffer-layer eddies using a field of white noise and show that the core flow is able to extract energy from these artificial fluctuations to organize turbulence eddies that maintain a physical turbulence mixing. Results for $Re_\tau = 640$, based on the channel half-height and on the friction velocity, are presented. Mean-velocity and root-mean-square statistics are compared to results from higher resolution simulations.

1. Introduction

Reynolds-Averaged Navier-Stokes modeling (RANS) and large-eddy simulation (LES) are the contemporary tools for modeling/simulating high-Reynolds-number flows. RANS, as a statistical approach, is particularly efficient for predicting mean velocity statistic and basic turbulence information. RANS models perform especially well near solid boundaries, since the underlying parameterizations are tuned for that class of flows. Unfortunately, RANS models do not generalize well for modeling the geometry-dependent scales. Large-eddy simulation, as a quasi-exact technique, is capable of simulating with fidelity the geometry-dependent scales of motion, so long as the LES filter scale lies in the inertial-range of the turbulence. For high-Reynolds-number flows, maintaining an inertial-range filter scale becomes difficult, particularly near walls where kinematic constraints restrict the energy-containing-range of the turbulence to progressively higher wavenumbers.

The apparent synergy between strengths of RANS and LES has been recognized by a number of authors. Speziale (1998) motivated their connection by recognizing that the RANS equations belong to a superclass of filtered Navier-Stokes equations. He proposed relating the subgrid turbulence diffusivity to the RANS eddy diffusivity using an appropriate transfer function. The RANS solution then becomes part of the subgrid model. Spalart, Jou, Strelets, and Allmaras (1997) outlined such a model which they called detached-eddy

simulation (DES). Their work and follow-on work by Nikitin et al. (2000) and Strelets (2001) shows that DES is able to yield good mean-field statistics and higher-order turbulence statistics as well.

Peltier, Zajaczkowski, and Wyngaard (2000) noted that the formal connection between RANS and LES is the convergence of an ensemble average (filtered over a characteristic volume) and a volume average of a flow field for averaging volumes that are large relative to the energy-containing-range length scale of the turbulence. They proposed a related hybrid RANS/LES technique that populates a stationary RANS field with turbulence scales.

Baggett (1998) concluded hybrid RANS/LES models are unlikely to work in near-wall regions of a flow because they cannot maintain a physical near-wall cycle for the turbulence. Similarly, Nikitin et al. (2000) describe "a danger zone" in which hybrid RANS/LES modeling of the near-wall flow cannot support turbulence. This work is a preliminary investigation of these observations using a modified version of the hybrid RANS/LES model proposed by Peltier et al. (2000).

2. Governing Equations

2.1 The Transport Budgets

The filtered, incompressible Navier-Stokes equations are solved for the resolvable scales of fully-developed turbulent channel flow. The flow is divergence free to enforce continuity. The equations are

$$\tilde{u}^r_{i,t} + (\tilde{u}^r_i \tilde{u}^r_j)_{,j} = -\tilde{p}^r_{,i} + \frac{1}{\mathrm{Re}_\tau}\tilde{u}^r_{i,jj} - \tilde{\tau}^{SGS}_{ij,j} - 1 \quad \text{and} \quad \tilde{u}^r_{i,i} = 0 . \tag{1}$$

The superscript "r" refers to "resolvable scale". The capping tilde is used to denote a variable with both mean and fluctuating parts. The "-1" on the right side is the mean pressure gradient nondimensionalized on the channel half-height and on the friction velocity. The pressure gradient term on the right side of (1) is the deviation from the mean gradient. Re_τ is the appropriate Reynolds number. Noslip conditions are enforced at the lower and upper walls of the channel. Wall functions are not used. The streamwise and cross-stream directions are periodic.

2.2 Turbulence modeling

The deviatoric part of the subgrid stress is modeled using eddy-diffusion,

$$\tilde{\tau}^{SGS}_{ij} - \frac{1}{3}\tilde{\tau}^{SGS}_{kk}\delta_{ij} = -\tilde{\nu}_T 2\tilde{s}^r_{ij} . \tag{2}$$

A modified Smagorinsky formulation is used to define the eddy diffusivity, $\tilde{\nu}_T$. The eddy diffusivity is the product of the characteristic velocity scale and the characteristic length scale of the turbulent flow. When the filter scale of a

computational model is suitably coarse, RANS modeling provides the appropriate diffusivity. Using ℓ and $q = \sqrt{\overline{u_i u_i}}$ to denote the RANS length and velocity scales, one writes

$$\overline{\nu}_T^{RANS} = \ell q \,. \tag{3}$$

The overbar denotes an ensemble-mean value and u_i is the fluctuating part of $\tilde{u}_i$. Note, we have absorbed the familiar coefficient $c_\mu = 0.09$ in our definition of ℓ and have embedded the standard damping coefficient in ℓ as well. For filter widths in the inertial range of the turbulence, one can define the characteristic scales Δ and v to write

$$\tilde{\nu}_T = \Delta \mathrm{v} \,. \tag{4}$$

The Smagorinsky subgrid model has been used extensively for inertial-range modeling. It relates v to the strain-rate invariant of the resolved flow, $\mathrm{v} = \Delta \tilde{S}$ where $\tilde{S} = 2\sqrt{\tilde{s}_{ij}^r \tilde{s}_{ij}^r}$, and uses the characteristic linear dimension of the local grid-cell volume to define Δ. Again, our notation absorbs the Smagorinsky coefficient, $c_s = 0.065$, and near-wall damping terms in Δ. Equations (3) and (4) define the limits of our hybrid RANS/LES model. We propose a simple blending between them to accommodate energy-containing-range modeling.

2.3 Energy-Containing-Range Modeling

The contraction defining the resolvable-scale strain-rate invariant has two components,

$$\overline{\tilde{s}_{ij}^r \tilde{s}_{ij}^r} = S_{ij} S_{ij} + \overline{s_{ij} s_{ij}} \,, \tag{5}$$

a mean strain-rate contribution, S_{ij}, and a contribution from the fluctuating strain-rate, s_{ij}. The mean-strain-rate part scales with the RANS length and velocity scales, so

$$S^2 \equiv S_{ij} S_{ij} \sim \left(\frac{q}{\ell}\right)^2 . \tag{6}$$

Inertial-range arguments for $\Delta << \ell$ show

$$s^2 \equiv \overline{s_{ij} s_{ij}} \sim \left(\frac{\Delta}{\ell}\right)^{-4/3} \left(\frac{q}{\ell}\right)^2 , \text{ so } S_{ij} S_{ij} << \overline{s_{ij} s_{ij}} \text{ for } \Delta << \ell \,. \tag{7}$$

Equation (7) says that when a flow field is resolved well, the mean strain-rate contributes minimally to the mean eddy diffusivity. Direct interactions with the mean flow are weak and the fluctuating field maintains the turbulence diffusion. Similar scaling, considering $\tilde{\nu}_T$ to be a property of a turbulent fluid, shows that

turbulence diffusion by the largest scales is dominated by direct interactions with the mean-strain-rate field,

$$\overline{s_{ij}s_{ij}} \sim \left(\frac{\Delta}{\ell}\right)^{-4}\left(\frac{q}{\ell}\right)^{2}, \text{ so } S_{ij}S_{ij} >> \overline{s_{ij}s_{ij}} \text{ for } \Delta >> \ell \ . \qquad (8)$$

Results (7) and (8) have important implications to modeling. The fine-grid limit is insensitive to the mean flow but requires reasonable modeling of turbulence fluctuations. No additional modeling is needed for this range, since traditional LES subgrid models are already adequate. The coarse mesh limit is sensitive to mean-flow parameterizations though insensitive to details of the evolving fluctuations. We infer that the well known failure of the Smagorinsky model in the coarse mesh limit comes from its interaction with the mean flow. Zajaczkowski and Peltier (Reg. Paper #68 of these proceedings) propose a correction to the Smagorinsky model based on accommodating the RANS prediction of the mean time scale. We use their corrected Smagorinsky model:

$$\tilde{v}_T = \tilde{v}_T^{SMAG} + T(\ell,\Delta,\eta)(\bar{v}_T^{RANS} - \bar{v}_T^{S}) \text{ where } \bar{v}_T^{S} = (c_s\Delta)^2\left(2\sqrt{S_{ij}S_{ij}}\right). \quad (9)$$

$\bar{v}_T^{S}$ represents the incorrect part of the Smagorinsky model, it's application to the mean flow, that is replaced by a correction from RANS. The correction imposes the proper time scale. The transfer function is $T(\ell,\Delta,\eta) = (\Delta/\ell)^2$.

3. Numerical Method

A finite difference discretization of Eq. (1) with discretized boundary conditions and turbulence modeling is solved. The solution procedure follows the fractional step approach outlined by Rai and Moin (1991); however, a linear blending of second-order accurate weighted-average central differencing with first-order accurate upwind differencing is used for the nonlinear advection terms for values of the transfer function greater than 0.9. This range was chosen by numerical experiment emphasizing the need to support turbulence scales of motion while retaining stability for very coarse grids. Explicit dependence of the blending on cell Peclet number was not used. The code was validated based on the previous study by Peltier et al. (2000) and based on comparisons to other experimental and numerical data.

4. Numerical Results

Our test problem is fully-developed channel flow at $\mathrm{Re} = 640$, based on the friction velocity and on the channel half height. This configuration is a common test case because of its geometric and flow condition simplicity; however, it imposes all of the difficulties associated with modeling near-wall flows. Our domain size is $2\pi \times \pi \times 2$, similar to cases studied by Moin & Kim (1982). Our grid resolution is varied from fine, $42\times23\times65$, to coarse, $14\times7\times17$. Intermediate resolutions are $14\times23\times33$, $14\times23\times17$, $14\times15\times17$, and

$14 \times 7 \times 33$. Hyperbolic tangent stretching is used in the wall-normal direction, our z coordinate. The near-wall spacing is prescribed to give $y^+ = 1$ at the second grid point. Equal spacing is used in the streamwise (x) and cross-stream directions (y). The mean statistics for this case are stationary, so the RANS input to (9) is sampled *apriori* to increase code efficiency.

Our parametric variations of grid resolution in the streamwise and wall-normal directions show that the accuracy of our predictions is affected by resolution, however, turbulence fluctuations are supported for cross-stream resolutions greater than 7. Considering the cross-stream resolutions, one sees that the dimensionless grid spacing in the cross-stream direction of the fine resolution case is 0.14, a value that can support turbulence based on observations by Nikitin et al. (2000), while the cross-stream spacing of coarse resolution case is 0.45, well within the "danger zone" outlined by Nikitin et al. (2000) for which turbulence cannot be supported.

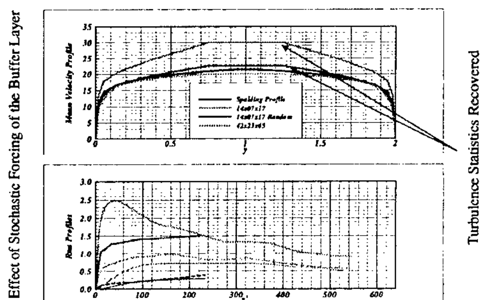

Figure 1 Mean velocity and rms profiles

Figure 1 shows that without special intervention, turbulence in our coarse resolution case is suppressed, as expected, leading to a nonphysical bulge in the mean velocity profile and zero turbulence fluctuations.

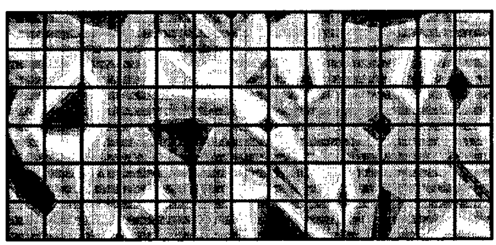

Figure 2 White noise applied to buffer layer, 10% turbulence intensity.

Since the buffer layer is predominantly decoupled from the core flow with the exception of isolated bursting events, we postulate that the effects of an under-resolved buffer-layer can be modeled using white noise of sufficient intensity. Figure 2 presents a white noise field used in this study. Ten percent turbulence intensity is arbitrarily chosen. The core flow responded with organized large-scale structures correcting the lost turbulence diffusion. Figure 1 shows that the mean-velocity profile computed from the coarse resolution case forced by buffer-layer noise recovers the Spalding profile as hoped.

5. Conclusions

The results of this work independently confirm the observations of Nikitin et al. (2000) that a "danger zone" exists in hybrid RANS/LES modelin g. Our hybrid model is markedly different than the DES they used showing that this "danger zone" is general. We show that a simple model bursting events from the underresolved buffer layer is able to sustain turbulence in the core flow. This result provides one route for constructing an hybrid RANS/LES modeling capability that is truly insensitive to grid resolution.

Future work will involve estimating bounds for the required turbulence intensity needed to model under-resolved buffer layer events.

Acknowledgments

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